Geometric Sequence

Lesson Vocabulary
- geometric sequence
- common ratio
- geometric mean

Objectives To identify mathematical patterns found in a sequence
To use a formula to find the nth term of a sequence

A sequence is a set of numbers related by a common rule. All sequences start with an initial term. In a **geometric sequence**, the ratio of any term to the previous term is constant. This constant ratio is called the **common ratio** and is denoted by $r\ (r \neq 1)$.

The **geometric mean** of two positive numbers x and y is $\sqrt{xy}$.

take note

Key Concept Geometric Sequence

A **geometric sequence** with a starting value a and a **common ratio** r is a sequence of the form

$$a, ar, ar^2, ar^3, \ldots$$

A recursive definition for the sequence has two parts:

$a_1 = a$ initial condition
$a_n = a_{n-1} \cdot r$, for $n > 1$ recursive formula

An explicit definition for this sequence is a single formula:

$a_n = a_1 \cdot r^{n-1}$, for $n \geq 1$

Essential Understanding In a *geometric sequence*, the ratio of any term to its preceding term is a constant value.

Sequences

Arithmetic Sequence

Sequence A

2, 4, 6, 8, ...

3, 6, 9, 12, ...

4, -1, -6, -11, ..

common difference

Geometric Sequence

Sequence B

1, -3, 9, -27, ...

3, 6, 12, 24, ...

100, 20, 4, ...

common ratio

Geometric Sequence

In general, when 0 is the starting position for the initial term a of an arithmetic sequence

The explicit rule is $f(n) = ar^n$.

and the recursive rule $f(0) = a$

$$f(n) = r \cdot f(n-1)$$

n	0	1	2	3	4	⋯	$j-1$	j	⋯
$f(n)$	3	6	12	24	48	⋯	$ar^{(j-1)}$	ar^j	⋯

When 1 is the starting position of the initial term, the sequence has the

The explicit rule is $f(n) = ar^{n-1}$.

and the recursive rule $f(1) = a$

$$f(n) = r \cdot f(n-1)$$

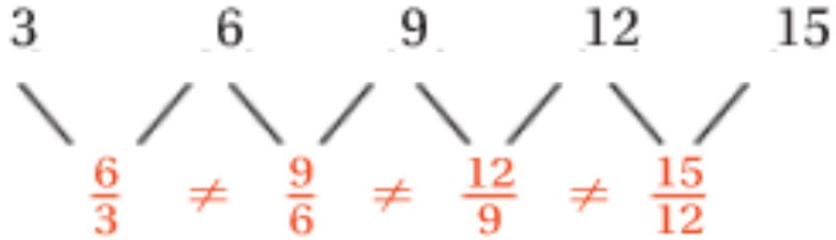
n	1	2	3	4	5	⋯	$j-1$	j	⋯
$f(n)$	$\frac{1}{25}$	$\frac{1}{5}$	1	5	25	⋯	$ar^{(j-1)}$	ar^j	⋯

Geometric Sequence

Identifying Geometric Sequences

B **3, 6, 9, 12, 15, . . .**

Find the ratio between consecutive terms.



Geometric Sequence

Objective To define, identify, and apply geometric sequences

C $3^5, 3^{10}, 3^{15}, 3^{20}, \ldots$

Use the properties of exponents to simplify the ratios of successive terms.

$$3^5 \quad 3^{10} \quad 3^{15} \quad 3^{20}$$

$$\frac{3^{10}}{3^5} = \frac{3^{15}}{3^{10}} = \frac{3^{20}}{3^{15}} = 3^5$$

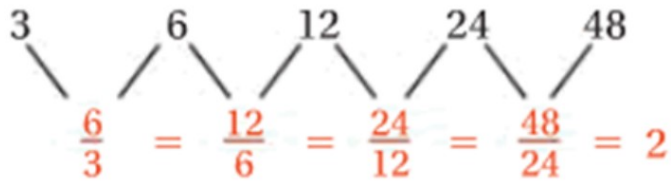
Geometric Sequence

Identifying Geometric Sequences

Is the sequence geometric? If it is, what are a_1 and r?

A **3, 6, 12, 24, 48, . . .**

Find the ratios between consecutive terms.



The common ratio is 2. The sequence is geometric with $a_1 = 3$ and $r = 2$.

Geometric Sequence

What are the indicated terms of the geometric sequence?

A **the 10th term of the geometric sequence 4, 12, 36, . . .**

The first term a_1 is 4. The common ratio r is $12 \div 4 = 3$.

$a_n = a_1 r^{n-1}$ Use the explicit formula.

Geometric Sequence

What are the indicated terms of the geometric sequence?

B **the second and third terms of the geometric sequence 2, ■, ■, −54, . . .**

The first term a_1 is 2. The fourth term a_4 is -54.

The explicit rule is $f(n) = ar^{n-1}$.

Geometric Sequence

In an arithmetic sequence, recall that the middle term of any three consecutive terms is the arithmetic mean of the other two terms.

The **geometric mean** of two positive numbers x and y is $\sqrt{xy}$.

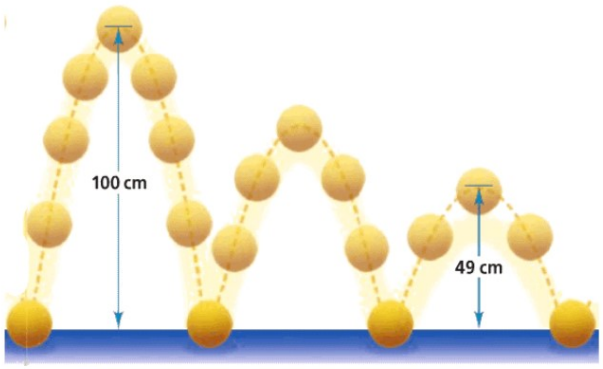
Note that the geometric mean is positive by definition. While there are two possible values for the missing term in the geometric sequence 3, ■, 12, . . . , there is only one geometric mean. The geometric mean is one possible value to fill in the geometric sequence. The opposite of the geometric mean is the other.

Got It? Ⓐ The 9th and 11th terms of a geometric sequence are 45 and 80. What are possible values for the 10th term?

Geometric Sequence

Solving a Real-World Problem Involving Geometric Sequences

Physics When a ball bounces, the heights of consecutive bounces form a geometric sequence. What are the heights of the 4th and 5th bounces?



$$a_n = a_1 r^{n-1}$$

Mars Curoisity Rover

Geometric Sequence

Any Questions?

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Classwork:

Worksheet 12.2

take note

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